NONSTATIONARY PROCESSES IN STARTING STRONGLY UNDEREXPANDED JETS

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Results of the investigation of nonstationary efflux of argon by the electron-beam-sounding method are presented in [1]. Comparing the regularities obtained in that paper for the front motion of material during efflux from a nozzle with computations [2] for nonstationary expansion from a spherical source and the experimental results in [3] permitted clarification of the singularities of the influence of counter pressure and the temperature factor in jet expansion. The density distribution in nonstationary nitrogen and argon jets is obtained in this paper and study of the regularities of the front motion of the escaping gas is continued.

§1. The efflux of nitrogen and argon was from a sonic nozzle of diameter $d_* = 0.25$ mm at the initial pressure $p_0 = 7$-8 atm, the pressure of the surrounding space $p_\infty = (1.5\text{-}2)\cdot 10^{-5}$ mm Hg, and the temperature $T_0 = T_\infty = 300$°K.

The sounding electron beam had the following parameters: voltage E = 15 kV, current I = 100 μA, beam radius $r_0 = 0.5$ mm, spacing between the gun and the collector L = 525 mm, and collector-entrance aperture $\emptyset = r_0/L \approx 0.5°$.

The values of the experimentally determined absorption coefficients of the electron gun were

$$\text{in Ar}\quad \varkappa_{\mathrm{Ar}} = (0.47 \pm 0.05)10^6\ \mathrm{cm^2/g},$$
$$\text{in N}_2\quad \varkappa_{\mathrm{N_2}} = (0.39 \pm 0.05)10^6\ \mathrm{cm^2/g}.$$

The absorption signals were recorded at spacings $80r_*$ to $2000r_*$ along the axis $80r_*$ to $800r_*$ on both sides of the stream axis. Oscillograms analogous to those represented in [1] were obtained during the experiment.

Statistical processing of the experimental data on the front motion of escaping gas was performed by least squares. Under the given experimental conditions, the empirical equations of front motion, determined in the form of quadratic binomials have the form

$$\mathrm{N_2}\quad t = 5x + 0.11x^2; \tag{1.1}$$
$$\mathrm{Ar}\quad t = 9x + 0.16x^2 \tag{1.2}$$

(x in cm; t in μsec; the coefficients have the appropriate dimensionality).

A program for numerical integration of the Abel equation on the HP-9810 electronic computer was compiled to compute the density field from the integral absorption data. Since the integral absorption curves inserted in the program were obtained from experimental data of a large number of tests, the computed density fields are averaged over a series of experiments of the same type. The error in the integral absorption curves was 20% for a confidence probability of 0.95; the error in the numerical integration did not exceed 1%, and the total error in determining the density is 25-30%.

The density distributions in different sections of nonstationary argon and nitrogen jets, respectively, are presented in Fig. 1a, b for different times [a) Ar: 1) 60 μsec, 2) 150 μsec, 3) stationary; b) N_2: 1) 40 μsec, 2) 75 μsec, 3) 150 μsec, 4) stationary]. The data of Fig. 1 yields a conception of the density drop with distance from the nozzle exit, the jet expansion, and the time development of the process.

An analysis of the data represented and their comparison with the experimental results in [1, 3] and a theoretical analysis [2] permitted the clarification of similarity parameters of pulsed jets and the space—time regularities of a nonstationary jet flow.

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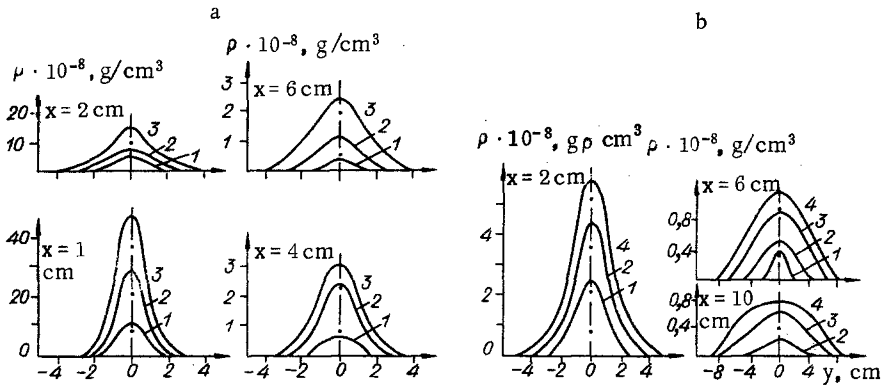

Fig. 1

§2. It is known that the governing parameters for stationary jets are the radius of the nozzle critical section r_*, the off-design of the jet $N = p_0/p_\infty$, the temperature factor T_0/T_∞, the numbers Re, Kn, M, and the ratio of the specific heats γ [4]. In describing nonstationary jets it is necessary to introduce a parameter including the time dimensionality. The characteristic stream velocity can be such a parameter.

The value $c_* = c_0\sqrt{[2/(\gamma+1)]}$ is used as the characteristic velocity in analyzing the experiments in [1, 3]. A theoretical analysis of nonstationary expansion from a spherical source [2] showed that the velocity of nonstationary expansion in a vacuum $u_m = [2/(\gamma - 1)]\cdot c_0$ can be used as the characteristic velocity for the front motion of an escaping gas. A change in the parameters in the later stages of jet formation can also characterize the velocity of stationary expansion in a vacuum $u_s = \sqrt{[2/(\gamma - 1)]}\cdot c_0$.

Similarity parameters, eliminating the dependence of the empirical front-motion equations (1.1) and (1.2) on the gas type and the efflux modes, are needed for an analysis and generalization of the experimental results on the front motion of the escaping gas along the stream axis.

An analysis of the front motion of the escaping gas along the stream axis in parameters, including different characteristic velocities, showed that the best generalizations in the gas type γ for identical efflux modes are similarity parameters including the maximum velocity of nonstationary expansion into a vacuum $u_m = [2/(\gamma - 1)]\cdot c_0$. The front-motion equations for an escaping gas under conditions of the present experiments (1.1) and (1.2) hence have the form

$$\mathrm{Ar}\ T_m = 0{,}86X + 1.9X\cdot 10^{-4}X^2; \tag{2.1}$$

$$\mathrm{N_2}\ T_m = 0{,}88X + 2.35X\cdot 10^{-4}X^2, \tag{2.2}$$

where

$$X = x/r_*;\ T_m = tu_m/r_*.$$

The front-motion equations under the experimental conditions in [3] acquire the following form in the coordinates mentioned:

$$\mathrm{Ar}\ T_m = 0.55X^{1{,}92}; \tag{2.3}$$

$$\mathrm{N_2}\ T_m = 0.7X^{1{,}85}. \tag{2.4}$$

As is seen from Fig. 2, the discrepancy between the experimental values on front motion in Ar and N_2 in the coordinates mentioned does not exceed 5-10%, while the discrepancy between the values on motion trajectories in nitrogen and argon reaches 80-100% when using the dimensionless time parameters $T_* = tu_*/r_*$ or $T_s = tu_s/r_*$, including the other characteristic velocities.

The analysis of (2.1) and (2.2) permits clarification of the interesting singularity of the front motion for the escaping gas under the present experimental conditions. Near the nozzle exit the front velocity exceeds the maximum nonstationary-efflux velocity into a vacuum [5] by 10-12%. Separation of the highly energetic molecules during efflux and growth of the stream kinetic energy because of condensation may be the reason for the anomalous increase in the front velocity recorded in the experiments.

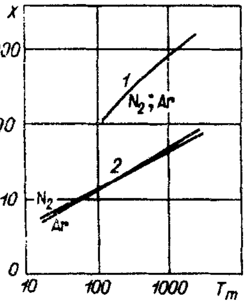

Fig. 2

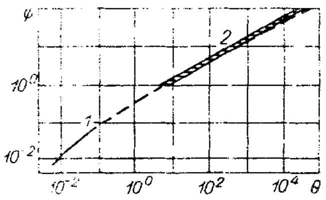

Fig. 3

According to [6, 7], the concentration of the condensed particles can reach $\alpha \approx 10^{-1} + 10^{-2}$ in Ar and $\alpha \approx 10^{-2} + 10^{-3}$ in N_2 under conditions close to the present experiment ($p_0^2 d_* \approx 1$ atm^2•cm). The stream velocity may hence grow 10-15% for Ar and 8-12% for N_2 [7].

Such comparisons are understandably only approximate since gas expansion during a nonstationary efflux is quite different from the expansion of the stationary stream in [6, 7].

The difference between the front motion for an escaping gas under the conditions of this paper and the front motion under the conditions in [3], which is graphically visible in Fig. 2 [1) this paper, 2) the paper [3]], can be analysed taking account of the difference between the governing parameters of the investigated jets.

It is seen from Table 1 that the modes studied differ substantially in both off-design N and temperature factors $Z = T_0/T_\infty$, and in the spacings from the nozzle exit X, measured in calibers, which were investigated.

The similarity coordinates generalizing the data obtained in the different modes should be represented by monomials:

time similarity $K_t = T_m N^a Z^b Re_*^c$.

coordinate similarity $K_x = X N^a Z^b Re_*^c$.

A statistical analysis of the data in [3] showed that (2.3) and (2.4) remain without changes within the limits of variation of the parameters N and Z presented in the table; i.e., $a \sim 0$, $b \sim 0$ under the conditions in [3].

On the other hand, the data available in the literature [2, 4] indicate that the parameters N and Z can enter into the similarity criteria in the powers $a = -0.5$, $b = 0.5$.

Similarity parameters for the front motion of a gas escaping from a spherical source are obtained in the form $\xi = XN^{-0.5} \cdot Z^{-0.5}$, $\tau = T_m N^{-0.5} Z^{-0.5}$ in [2] devoted to a theoretical analysis of nonstationary expansion from a source. The front-motion for an escaping gas, obtained on the basis of [2], has the form

$$\tau = \xi + 0{,}5\xi^2.$$

The front-motion equations obtained in this paper have the following form in these parameters

$$\text{Ar}\ \tau = 0{,}86\xi + 3{,}8\xi^2; \qquad (2.5)$$

$$\text{N}_2\ \tau = 0{,}88\xi + 4{,}7\xi^2, \qquad (2.6)$$

and the equations in [3] become

$$\text{Ar}\ \tau = (1{,}25\text{—}1{,}8)\xi^{1{,}92}; \qquad (2.7)$$

$$\text{N}_2\ \tau = (1{,}4\text{—}2{,}1)\xi^{1{,}85}. \qquad (2.8)$$

Extrapolation of the empirical equations (2.5) and (2.6) to the values of the parameters ξ and τ corresponding to the conditions in [3] indicates the essential difference (~threefold) in the front location at fixed times in the parameters ξ and τ under different experimental conditions.

In analyzing the front motion of an escaping gas in the parameters

$$\Psi = X/\sqrt{N},\ \Theta = T_m/\sqrt{N}, \qquad (2.9)$$

TABLE 1.

Source	X	N	Z	Re_*
This paper	100—200	$4\cdot10^8$	1	$3\cdot10^4$
The paper [3]	1+100	50+100	6+12	10^3

which are analogous to the similarity parameters for stationary jets [4], the Eqs. (2.5) and (2.6) remain unchanged in the present experiments because $N = \rho_0/\rho_\infty$, while Eqs. (2.7) and (2.8) take the form

$$\mathrm{Ar}\ \Theta = (3.3—4.5)\Psi^{1.92}; \tag{2.10}$$

$$\mathrm{N}_2\ \Theta = (3.6—5)\Psi^{1.85}. \tag{2.11}$$

The nonuniqueness of the front-motion equations in [3] in the coordinates presented here and below [Eqs. (2.7), (2.8) and (2.10), (2.11) and Fig. 2] is in principle, and due to the difference between the coordinates under consideration and the similarity coordinates in [3].

Graphs of the empirical front-motion equations obtained in Ar and N_2 under different efflux conditions (the dashes are the extrapolation of the empirical equations of this paper to large values of the parameters Ψ and Θ); it is seen that the extrapolated curve 1 drops in the corridor of the values 2 which comprise the experimental data in [3] on these parameters generalized in the coordinates X and T_m.

Therefore, the empirical front-motion equations presented in Fig. 3 in the generalized similarity parameters (2.9) can be used to describe the front motion of nonstationary jet expansion of a gas from a sonic nozzle.

§3. The advancement of stream points corresponding to fixed values of the density is examined to analyze the process of density growth in the axial jet direction. The dependences obtained in such a manner also characterize the process of the density approaching the stationary values.

Experimental results on changes in the density distribution with time are analyzed in the dimensionless parameters X and $T_m = T_* = tc_*/r_*$, $T_m = tu_m/r_*$, $T_s = tu_s/r_*$.

The difference between the results obtained in Ar and N_2 did not exceed 5-10% when using the parameter T, while it was 30-40% when using the parameters T_* and T_m. This result indicates that, in contrast to front motion, the process of density growth along the axis is characterized by the stationary-expansion velocity in a vacuum.

The experimental results obtained showed that the total time of density growth at axial stream points after the front arrival varied weakly with distance from the nozzle exit. Hence, for further analysis of the density growth, the dependences obtained have been constructed in the coordinates X/X_{st} and $\overline{T}_s = T_s - T_s^{fr}$, where T_s^{fr} is the time for the front to reach the coordinate X and X_{st} is the stationary position of this density value. The curve generalizing the experimental results in these coordinates is presented in Fig. 4 (dark points are argon and light points are nitrogen). An analysis of the data obtained by least squares in the form of an exponential dependence showed that the process of axial stream-point propagation with given density values can be described with not more than 10% error by the equation

$$X/X_{st} = 1 - e^{-\overline{T}_s/400}. \tag{3.1}$$

Comparing the analytic curve (3.1) with the density-growth process at fixed stream points $\rho/\rho_{st} = f(\overline{T}_s)$ on which the flow irregularities exerted greater influence showed that this process can also be described in (3.1) with a ±30% error.

Therefore, the nature of the density growth in the axial stream direction after the arrival of the escaping gas front is independent of the distance from the nozzle exit and can be described by an exponential with the time constant $\overline{T}_s = 400 \pm 50$ or $\tau_* = (400 \pm 50)\cdot(r_*/c_0)\sqrt{(\gamma - 1)/2}$, independently of the species of gas.

To analyze jet expansion during its formation the time dependences of the location of the transverse-stream coordinates at which the density is $\rho = 0.1\rho_0$ were considered, where ρ_0

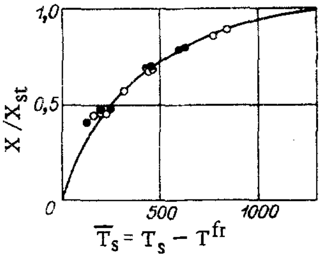

Fig. 4

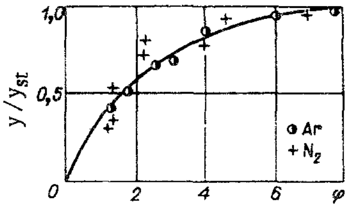

Fig. 5

are running values of the density on the axial line in this stream section. The experimental dependences obtained indicated that, in contrast to the nature of the density growth along the axis, this process alters the time scale sufficiently strongly depending on the distance from the nozzle exit. An approximate analysis determined that the time of density growth in the transverse direction increases approximately in proportion to the time the front reaches this stream section. Propagation of the transverse-stream points y/y_{st} is represented in Fig. 5 as a function of the dimensionless time parameter $\varphi = T_s/T_s^{fr}$ (y_{st} is the stationary location of the density value).

Processing the experimental results in the form of an exponential dependence showed that the process of jet expansion can be described by the relationship $y/y_{st} = 1 - e^{-\varphi/2}$ to 20% accuracy; i.e., the time constant of this process is $\tau_y = (2 \pm 0.4)t^{fr}$.

It is seen from a comparison of the nature of nitrogen and argon expansion that the nitrogen stream expands at large angles and with a steeper dependence of the density on the distance from the nozzle exit at distances to $(200\text{–}300)r_*$. At large distances the flow in both gases is complex.

The differences in the nature of nitrogen and argon expansion which were noted can be due to the influence of the molecule rotational energy on nitrogen expansion. The rotational relaxation of nitrogen in a jet expanding freely into a vacuum out of a sonic nozzle is computed in [8]. The author obtained that the rotational temperature T_r is "frozen" at distances on the order of $(40\text{–}50)r_*$ ($T_r \approx 10$°K) for $p_0 d_* = 240$ mm Hg and the number of collisions $Z = 10$ needed to build up equilibrium, and the influence of the rotational energy on the behavior of the gasdynamic parameters if felt at somewhat greater distances (because of the significant energy delivery during individual R–T collisions).

These results are in qualitative agreement with the results of this paper, which indicates that the nature of the nitrogen stream expansion does not differ from the expansion of a monatomic gas stream at distances greater than $(200\text{–}300)r_*$ for $p_0 d_* \simeq 1300$ mm Hg.

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